Math 210 - Quiz 1 (Fall 2010)

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1. (10 points each) Prove the following claims:

i- $\{a_n\}_{n=1}^{\infty}, \{b_n\}_{n=1}^{\infty}$ are two sequences converging to L and M respectively then $a_n + b_n \to L + M$. Moreover, show that the conclusion holds even if we allow L and M to take the values $+\infty$ and $-\infty$ as long as $L + M$ is not of the form $\infty - \infty$.

ii- Prove that $\frac{1}{a_n} \to \frac{1}{L}$ as long as $L \neq 0$. Moreover, show that if we interpret $\frac{1}{\infty}$ and $\frac{1}{-\infty}$ as equal to 0 the result still holds.

iii- Give an example of a sequence $\{a_n\}_{n=1}^{\infty}$ such that $a_n \to 0$ but $\frac{1}{a_n} \nrightarrow \infty$ or $\frac{1}{a_n} \nrightarrow -\infty$.

$(a_n \neq 0)$

2. (10 points) Can there be a cauchy sequence of rationals which does not converge to a rational? Justify your answer.

3. (6 points) Prove that $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges.

4. (10 points each) Answer, with justification, the following questions:

i- Is there a sequence whose range has no limit points?

ii- Is there a convergent sequence whose range has no limit points?

iii- Is there a cauchy sequence whose range has no limit points?

iv- Is there an unbounded sequence whose range <u>does</u> have a limit point?

5. (7 points each) Give an example of a collection of open sets $\{O_n\}_{n=1}^{\infty}$ such that

i- $\bigcap_{n=1}^{\infty} O_n$ is not open.

ii- $\bigcap_{n=1}^{\infty} O_n$ is not closed.